P-and-T–VIOLATION TESTS WITH POLARIZED RESONANCE NEUTRONS

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Abstract: The enhancements of CP-violating effects in resonance neutron transmission through polarized targets are studied for 2 possible versions of experiment. The importance is stressed of error analysis and of pseudomagnetic effects' compensation.

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1. Introduction. It was shown [1-3] about 15 years ago that CP-violation effects in transmission of polarized neutrons through the polarized target might be enhanced in the vicinity of p-resonances by 5-6 orders of magnitude. Originally it was suggested to measure the difference in transmission of neutrons with spins parallel (N_+) and antiparallel (N_+) to the vector $\vec{k}_n \times \vec{I}$ ($\vec{k}_n$ and $\vec{I}$ are the neutron momentum and the target spin):

$$\eta_T = \frac{N_+ - N_-}{N_+ + N_-} \approx 2\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \tag{1}$$

Here N_+ and N_- are the numbers of neutrons with the corresponding helicities transmitted through the polarized target sample, σ_+ and σ_- are the corresponding total neutron cross-sections. However, it was pointed [4] that without the special precautions the nuclear pseudo-magnetic precession of neutron spin together with the precession induced by the P-violating interactions would give rise to numerous effects camouflaging the CP-violating ones. As a possible remedy of this nuisance it was suggested [4] to compensate the nuclear pseudo-magnetic field by the external magnetic field in order to nullify the neutron spin rotation angle ϕ. However, in order to measure the CP-violating interaction with the reasonable accuracy (about 10^{-4} of the P-violating one) it was necessary to check the spin rotation angle with the precision of about 10^{-7} rad [4].

In order to circumvent the above difficulties Stodolsky [5] suggested to measure the difference $N_{+-} - N_{-+}$, where N is the number of neutrons transmitted through the target and the subscript indices mean the neutron helicity before and after the transmission. Consider the polarized neutron scattering amplitude of the form:

$$f = A + p_t B \cdot (\vec{s}_n \cdot \vec{I}) + C \cdot (\vec{s}_n \cdot \vec{k}_n) + p_t D \cdot \vec{s}_n \cdot [\vec{k}_n \times \vec{I}] \tag{2}$$

where $\vec{s}_n$ is neutron spin, p_t is the target degree of polarization, A and B are the spin-independent and spin-dependent parts of the strong interaction amplitude, D is the P- and CP-violating interaction amplitude, respectively. The term C contains contributions from both weak P-violating and strong interaction (from the term of the type $(\vec{s}_n \cdot \vec{k}_n)(\vec{k}_n \cdot \vec{I})$ in scattering amplitude -see e.g. [6]).

Stodolsky demonstrated that the difference

$$N_{+-} - N_{-+} \sim Im(DB^*) \tag{3}$$

is free from the above camouflaging effects. It is well-known that in order to improve the accuracy it is preferable to measure the relative values, i.e. to normalize the above difference. Although Stodolsky never bothered to introduce this normalization, it seems natural to consider the ratio:

$$T = \frac{N_{+-} - N_{-+}}{N_{+-} + N_{-+}} \tag{4}$$

A few years later Serebrov [7] suggested to measure the quantity:

$$X = \frac{(N_{++} - N_{--}) + (N_{+-} - N_{-+})}{(N_{++} - N_{--}) - (N_{+-} - N_{-+})} \quad (5)$$

One can easily see that

$$X = 1 + 2\frac{N_{+-} - N_{-+}}{(N_{++} - N_{--}) - (N_{+-} - N_{-+})} \quad (6)$$

The actual CP-violating effect causes the deviation of X from unity. Therefore the actually measured quantity $\tilde{X}$

$$\tilde{X} = \frac{N_{+-} - N_{-+}}{(N_{++} - N_{--}) - (N_{+-} - N_{-+})} \quad (7)$$

is simply the one suggested by Stodolsky, but normalized in a rather odd manner.

The main point is that up to now nobody cared to do the analysis of the energy dependence of the quantities T or $\tilde{X}$ in the manner it was done for the originally considered CP-violating quantity η_T in refs. [1-3]. Indeed , all the quantities in the numerators and denominators of T and $\tilde{X}$ contain various combinations of real and imaginary parts of all the four amplitudes (A, B, C and D) in Eq. (2). Most of them show a rather complicated energy dependence (see e.g. [3,6,8]) in the resonance region. Some of them not only vary in magnitude, but even change their sign. This means that up to now one does not know whether the suggested values T and $\tilde{X}$ are really enhanced and what is the magnitude of this enhancement, if any. Investigation of these problems is the main point of our present publication. For the time being we are not going to consider the false effects arising from the difference of the polarizing and the analyzing power of polarizer and analyzer. We shall also restrict ourselves with cases of "ideal geometry" when the incident beam polarization is either parallel or anti-parallel to the neutron momentum.

2. Analysis of T. In order to obtain the expressions for the relative quantities of interest in terms of the energy-dependent complex amplitudes A, B, C and D of Eq. (2), one might use the method developed in ref. [9]. Introducing the spin density matrix and the evolution operators of ref. [9], one obtains the expression for T:

$$T = 2\frac{Im(DB^*)}{|D|^2 + |B|^2} \quad (8)$$

The expressions for complex amplitudes $D(E)$ and $B(E)$ are obtained using the methods of ref. [3] (see also [6], [8]). The main contribution to the T-noninvariant amplitude D in the vicinity of the $p_{I+1/2}$ -resonance comes from the term coupling this resonance with the corresponding $s_{I+1/2}$-resonance:

$$D \approx \frac{\gamma^s_{I+1/2}}{(E - E^s_{I+1/2}) + \imath\Gamma_s/2}V_T\frac{\gamma^p_{I+1/2}}{(E - E^p_{I+1/2}) + \imath\Gamma_p/2}. \quad (9)$$

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In the optimal cases (like La target) these $s_{I+1/2}$ and $p_{I+1/2}$ resonances contribute equally to the strong amplitude B in this energy region. Taking into account all the other resonances would only lead to some numerical changes, while the general qualitative picture would be the same. Therefore we consider:

$$B \approx \frac{\gamma^s_{I+1/2}\gamma^s_{I+1/2}}{(E - E^s_{I+1/2}) + \imath\Gamma_s/2} + \frac{\gamma^p_{I+1/2}\gamma^p_{I+1/2}}{(E - E^p_{I+1/2}) + \imath\Gamma_p/2}, \tag{10}$$

Inserting these expressions into eq. (8) we see that the quantity $T(E)$ in the vicinity of the p-wave resonance energy E_p is:

$$T(E) \approx -2\frac{\gamma^p_{I+1/2}}{\gamma^s_{I+1/2}} \cdot \frac{V_T \cdot \Gamma_p}{(E - E_p)^2 + \Gamma_p^2/4} \tag{11}$$

Here Γ_p stands for the p-resonance total width, while V_T is the matrix element of CP-violating interaction causing the transition between the p- and s-resonance states. Further on in our numerical calculations we shall assume the ratio of the CP-violating interaction strength to the P-violating one to be 10^{-4} (i.e. $V_T/V_P = 10^{-4}$). The quantities $\gamma^{s,p}_J$ stand for the neutron width amplitudes of the s- and p-resonances with spin $J = I + 1/2$. The sign of the effect is defined by the signs of γ's and V_T. For the sake of simplicity we shall choose them in our numerical calculations so that the net effect is positive.

We observe in Eq. (11) the resonance enhancement of the effect typical for all the symmetry-breaking effects in nuclear reactions (see [3,8]). In order to see explicitly the "dynamical enhancement", which is also typical for these effects, one might cast the value of T in this maximum in the following form:

$$T\left(E = E^p_{I+1/2}\right) \approx \frac{\gamma^p_{I+1/2}}{\gamma^s_{I+1/2}} \cdot \frac{V_T}{d} \cdot \frac{d}{\Gamma} \tag{12}$$

Here d and Γ stands for the average resonance spacing and total width. It is instructive to remind that the corresponding expression for the maximal value of the quantity η_T obtained in [1-3] was:

$$\eta_T(E = E_p) \approx \frac{\gamma^s}{\gamma^p} \cdot \frac{V_T}{d} \cdot \left(\frac{d}{\Gamma}\right)^2 \tag{13}$$

Comparing Eqs.(12) and (13), one can see in both cases the presence of the dynamical enhancement factors $V_T/d \approx F_T \cdot 10^3$ (F_T is the strength of the CP-violating interaction relative to the strong interaction one) and of the resonance enhancement factors $d/\Gamma \approx 10^3$ coming from the fact that the effect is proportional to the time $\tau \sim (1/\Gamma)$ spent by the incident neutron in the CP-violating field of the target. We also see the presence of the "entrance channel hindrance" factor (see [3,8]) $\gamma^p/\gamma^s \approx 10^{-3}$ typical for all the low energy scattering experiments with P-violation. However, the resonance enhancement factor enters the quantity η_T quadratically, while T contains it only linearly. Therefore

the net enhancement of the T quantity is only by a factor of 10^3 instead of the $10^5 \div 10^6$ factor in η_T.

These conclusions are illustrated in Fig. 1a, where the energy behavior of the quantity $T(E)$ is shown for the particular case of the famous La p-resonance at $E_p = 0.75$ eV.

Consider now a very important problem of the optimal choice of the target sample thickness. One should mind that in the case of η_T value, likewise in the case of the longitudinal polarization P caused by the P-violating weak interaction, the correct expression for the experimentally measured ratio can be written as follows (see e.g. [10], [3] and [11]):

$$\frac{N_+ - N_-}{N_+ + N_-} \approx \frac{\sigma_+ - \sigma_-}{2} \cdot x \cdot \rho \tag{14}$$

where x is the target sample thickness and ρ is the density of nuclei in this sample. Since the experimentally observed effect is linear in target thickness, it seems that one should choose the thickest target possible. However, the neutron countings $N_\pm$ decrease exponentially with x. Therefore the statistical relative error of measuring each N value

$$\frac{\delta N}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{N_0}} e^{x \rho \sigma / 2} \tag{15}$$

also increases exponentially with x (N_0 here stands for the number of polarized neutrons incident on the target). In order to find the optimal target thickness x_0 one should estimate the relative error σ_{η_T} / η_T of the quantity in the l.h.s. of Eq. (14) and define its minimum (by equating the x-derivative of the relative error to zero). In this way one obtains that the optimal thickness in the case of η_T quantity is $2\lambda = 2/\sigma\rho$ (here λ stands for the mean free path of the neutron in the target sample). It is only by choosing the optimal x_0 that one obtains the last line in Eq. (1).

The relative error of the quantity T looks more complicated. One can easily see that the main contribution to it comes from the relative error of the numerator in T:

$$\frac{\sigma_T}{T} \approx \frac{e^{\frac{Im(A)}{Im(f)} \frac{x}{\lambda}}}{\sqrt{2N_0}} \cdot \frac{|q|}{\sin(\theta)} \cdot \frac{1}{\sqrt{ch^2\left(\frac{Im(q)}{Im(f)} \frac{x}{\lambda}\right) - \cos^2\left(\frac{Re(q)}{Im(f)} \frac{x}{\lambda}\right)}} \cdot \sqrt{\frac{|D|^2 + |B|^2}{Im^2(DB^*)}} \tag{16}$$

Here λ is the neutron mean free path and the (complex) quantity q is defined as:

$$q = \sqrt{\frac{1}{4}\sin^2(\theta)B^2 + \frac{1}{4}\sin^2(\theta)D^2 + \frac{1}{4}(C + \cos(\theta)B)^2} \tag{17}$$

The angle between the target polarization and neutron momentum vectors is denoted as θ. The $\sin(\theta)$ behaviour of Eqs. (16) reflects the fact that the CP-violating term in the amplitude (2) is proportional to $\sin(\theta)$. Therefore irrespective of the value of D the CP-violating effects disappear for $\theta \approx 0$ and the relative error goes to infinity. The dependence of Eq. (16) on the target thickness x is complicated by the periodic $\cos^2$ oscillations. The physical origin of those oscillations is the pseudo-magnetic neutron spin

rotation, discussed in ref. [4] - the neutron spin performs about a hundred rotations per mean free path in the target sample. The explicit dependence of the relative error (16) on the target thickness is shown in Fig. 1b for the case of the same p-resonance in La. The total number of the polarized neutrons N_0 incident on the target was somewhat arbitrary chosen to be 10^{18}.

One can see from Fig. 1b that the first minimum of the relative error is located about $x \approx 10^{-2}\lambda$. However, a slight change of x increases the relative error by orders of magnitude, which makes the analysis of the experimental results practically impossible.

This forces us to return to our initial idea [4] of compensating the pseudomagnetic precession by the external magnetic field. This field can be formally taken into account by substituting $Re(B)$ in the initial Eq. (2) by:

$$Re(B') = Re(B) - H \tag{18}$$

Here H stands for the value of the external magnetic field. Since the "pseudo-magnetic" amplitude $B(E)$ is energy-dependent, we can do the compensation by, say, putting $ReB'(E) = 0$ at $E = E_p + \Gamma_p/2$. Fig. 1d shows the dependence of relative error on x with this compensation. As expected, all the oscillations of Fig. 1b disappear and the relative error shows a minimum at around $x \approx 2.5\lambda$.

However, the effect T itself depends on the value of $ReB(E)$ - see Eq. (8). Without the compensation $Re(B) \gg Im(B)$ (approximately by 3 orders of magnitude) and the dominant contribution to the denominators and numerators of Eq. (8) comes from it. If we do the above compensation, then $Im(B) \gtrsim Re(B)$ and the effect in the vicinity of p-resonance ($|E - E_p| \leq \Gamma_p$) can be expressed as:

$$T' \approx -2\frac{Re(D)}{Im(B)} \tag{19}$$

Taking into account the energy dependence of the amplitudes, we get:

$$T' \approx 4\frac{\gamma^p}{\gamma^s} \cdot \frac{V_T}{\Gamma_s} \cdot \frac{d(E - E_p)}{(E - E_p)^2 + \Gamma^2/4} \tag{20}$$

Therefore the effect now changes sign at around the resonance energy E_p and reaches at the points $E \approx E_p \pm \Gamma_p$ its maximal value:

$$T' \approx \frac{\gamma^p}{\gamma^s} \cdot \frac{V_T}{\Gamma} \cdot \frac{d}{\Gamma} \tag{21}$$

Comparing this result with Eqs. (11), (12), we see that the compensating magnetic field, besides removing the oscillations of the relative error, also produced a important increase of the value of T itself, giving an extra resonance enhancement factor $d/\Gamma \sim 10^3$. It also radically changed the energy-dependence of the effect. By comparing Eqs. (8) and (16)

we see that the relative error in the presence of compensation decreases by the same 3 orders of magnitude.

These conclusions are confirmed by the results of calculating the effect under conditions of complete compensation $Re(B'(E_p + \Gamma_p/2)) = 0$ - see Fig. 1c.

Thus we see, that our initial idea [4] of compensation the pseudomagetism turns out still to be quite productive. The only remaining point is to estimate the practically necessary accuracy of this compensation. Following [4], we still think that the practical way of controlling this accuracy is by measuring the neutron spin rotation angle $\phi = 2Re(B)/Im(f) \cdot x/\lambda$ around $\vec{I}$ after its transmission through the target sample. Fig. 2a shows the dependence of the effect $T(E = E_p + \Gamma_p/2)$ on the spin rotation angle (which serves as a measure of the applied compensating magnetic field).

Fig. 2b shows the same dependence for the relative error. We see that both the effect and its relative error are optimal for practically complete compensation ($\phi \approx 0$). The slight shift of optima to small positive ϕ is caused by the interference of contributions to the effect from the pseudo-magnetic rotation and rotation caused by the T-noninvariant field D. However, the relative error changes only by a factor of 2 - 3 when the rotation angle varies from 0^0 to 200^0. Thus the limitations on the accuracy of compensation are quite moderate from this point.

A more essential limitation might come from the fact that the energy dependence of the effect (and, to somewhat less extend, its maximal value) changes rapidly with increasing ϕ. In order to see this, one might compare the curves in Fig. 1c (corresponding to $\phi = 0^0$) and Fig. 1a, calculated without compensation.

Therefore we decided to formulate the problem of the compensation accuracy in a slightly different way: We assume that a reasonable value for the experimental energy resolution is $\Delta E \approx 10^{-2}$ eV and consider the practically reasonable accuracy $\Delta\phi$ of measuring ϕ as a free parameter. Then the rotation angle ϕ (and thus the compensating field H) should be chosen in such a way that energy maximum of the effect $T(E)$ should be shifted by less than ΔE while varying the rotation angle in the interval from $\phi - \Delta\phi$ to $\phi + \Delta\phi$. On performing a good deal of "computer experiments" we can state, that the accuracy $\Delta\phi = 5^0$ is quite sufficient from this point of view.

Thus we see, that the limitations on the accuracy of measuring the rotation angle in order to check the compensation of pseudo-magnetic rotation are quite tolerable.

3. Analysis of $\tilde{X}$. Consider now the quantity $\tilde{X}$. As already mentioned, it differs from T only by the normalization factor. Therefore it is also enhanced in the vicinity of the p-wave resonance. However the new normalization makes the effect itself (and not only its relative error) dependent both on the angle θ and on the target thickness x. Moreover, the rapid energy oscillations are superimposed on the resonance behaviour of the effect.

The character of these oscillations depend on the target thickness x in a very complicated way. For the sake of illustration we show in Fig. 3 the energy dependence of $\tilde{X}$, calculated for $x \approx 2.5\lambda$.

All this considerably complicates the analysis. It is difficult even to find a reasonable analytical approximation for $\tilde{X}$. In the case of thick target (for La resonance this means $x \gtrsim 15\lambda$) one can write:

$$\tilde{X} \approx -\frac{\sin^2(\theta)Im(DB^*)}{\sin^2(\theta)Im(DB^*) + 2Re\left(q(C^* + \cos(\theta)B^*)\right)} \qquad (22)$$

Eq. (22) shows that in the thick target limit the rapid oscillations of the effect disappear. This makes the analysis of its energy and θ dependence much easier. Consider now the θ dependence of the numerator and the denominator in $\tilde{X}$ separately.

The whole interval of θ values can be separated into two regions. In the first region one can neglect all the contributions to the denominator besides $2\cos(\theta)Re(qB^*)$. In this region $q \approx \frac{1}{2}B$ and

$$\tilde{X} \approx -\frac{\sin^2(\theta)}{\cos(\theta)}\frac{Im(DB^*)}{|B|^2}. \qquad (23)$$

One can see that in this region $\tilde{X} \sim -T$.

Consider now the relative error of $\tilde{X}$ in this range of θ. In analogy to the above T case, the main contribution to this error comes from the numerator. Therefore

$$\frac{\sigma_{\tilde{X}}}{\tilde{X}} \approx \frac{\sigma_T}{T}, \qquad (24)$$

This conclusion turns out to be valid even without the thick target approximation. Therefore the relative error of $\tilde{X}$ strongly oscillates with the variation of the target thickness. The necessity of the compensating external magnetic field is again obvious. Introducing this compensation, we again observe that the oscillations of $\sigma_{\tilde{X}}/\tilde{X}(x/\lambda)$ disappear, and it is possible to find the optimal target thickness (which is obviously $x \approx 2.5\lambda$).

In the first region of angles the value of the effect increases when θ approaches the critical point where the denominator of the effect equals zero. The relative error remains more or less constant. This does not mean, however, that it is better to make measurements closer to this critical point, because the absolute value of the error also increases. Therefore the accuracy of experimental observations remains practically the same.

The second region of angles is characterized by the inequality $\cos(\theta)Re(qB^*) \ll Re(qC^*)$ and is located in the vicinity of $\theta = \pi/2$. The width of this region depends on the incident neutron energy and on the magnitude of the compensating magnetic field. Without the compensation this width is $10^{-4} \div 10^{-7}$ rad. In case of full compensation it increases to a few degrees. It is important to note that this region contains the value of θ which turns the denominator into zero, while the effect formally increases to infinity. In this second region the denominator's contribution to the relative error dominates.

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Therefore the relative error continuously increases and becomes infinite in point where the denominator equals to zero. This means that one should not come too close to the values $\theta = \frac{\pi}{2}$ because of the finite angular divergence of any experimental beam. It is obviously practically impossible to estimate the actual accuracy of the measurements carried close to $\theta = \frac{\pi}{2}$.

In order to estimate the necessary accuracy of compensation by the external magnetic field, one should perform the same kind of analysis as in the case of T. It is obvious that the results of such an analysis would be essentially the same: the main limitation on the accuracy should again come from the rapid change of the effect's energy dependence. As in the case of T, the resulting limitations on $\Delta\phi$ accuracy are quite reasonable.

4. Summary

We can draw the following conclusions:

Analysis of the CP-violating effect's relative error is by no means less essential than analyzing the effect itself. One can always normalize the CP-noninvariant difference (3) dividing it by a very small quantity. However such a normalization would not increase the accuracy of the measurement.

The necessity to compensate the pseudomagnetic precession is caused essentially by the fact that without such a compensation the accuracy of measurement varies with target thickness in a practically uncontrollable way.

The compensation of the pseudomagnetic precession increases by 3 orders of magnitude not only the effect itself but also the accuracy of its measurement. The net enhancement in the vicinity of p-wave resonance with compensation reaches 6 orders of magnitude. The energy dependence of the effect changes drastically in the presence of compensation.

As a practical way to control the degree of compensation we suggest, following [3], to measure the rotation angle ϕ of neutron polarization around the target polarization vector. When ϕ varies between 0^0 and 200^0 the maximal value of the effect and its relative error varies not more than by a factor of $2 \div 3$. The most stringent restriction on the accuracy of measurement of this angle comes from the fact that the energy dependence of the measured effect strongly depends on the value of the compensating magnetic field. With this restriction in mind it seems sufficient to fix ϕ with the accuracy about 5^0.

The CP-noninvariant quantity X suggested for measurement in ref. [7] is shown to differ from Stodolsky's CP-noninvariant difference (3) practically only by the choice of normalization factor. This factor becomes zero in the vicinity of $\theta = \pi/2$ (the beam polarization orthogonal to the target one). Although the value of thus normalized effect tends to infinity, its relative error also tends to infinity in this range of θ values. In the remaining range of θ, where the normalizing factor exceeds the CP-noninvariant difference (3), the relative error depends on θ angle as $1/\sin(\theta)$. Although the value of the effect

in this range of θ angles behaves as $\tan(\theta)$ and strongly increases approaching the value $\theta \approx \pi/2$, the relative accuracy of its measurements (besides the small vicinity of $\theta \approx 0$) remains practically the same.

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1a

$T,\ 10^8$

$E,\ eV$

1b

$\frac{\sigma_T}{T}$

$\frac{x}{\lambda}$

2a

$T, 10^8$

ϕ, deg.

2b

$\frac{\sigma_T}{T}$

ϕ, deg.

1